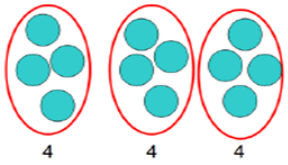
DIVISION

Stage 1 Group and share small quantities

Using objects, diagrams and pictorial representations to solve problems involving **both grouping and sharing**.

How many groups of 4 can be made with 12 stars? = **3**

Grouping



Sharing – 12 shared between 3 is 4 each

Example division problem in a familiar context:
There are 6 pupils on this table and there are 18 pieces of fruit to share between us. If we share them equally, how many will we each get? Can they work it out and give a division statement e.g. "18 shared between 6 people gives you 3 each."

Pupils should :

- use lots of practical apparatus, arrays and picture representations
- be taught to understand the difference between "grouping" objects (How many groups of 2 can you make?) and "sharing" (Share these sweets between 2 people)
- be able to count in multiples of 2s, 5s and 10s.
- find half of a group of objects by sharing into 2 equal groups.

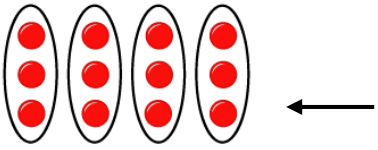
Key Vocabulary: share, share equally, one each, two each…, group, groups of, lots of, array
Key number skills needed for division at Stage 1:

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher
- Through grouping and sharing small quantities, pupils begin to understand, division, and finding simple fractions of objects, numbers and quantities.
- They make connections between arrays, number patterns, and counting in twos, fives and tens.

Stage 2 Group and share, using the ÷ and = sign

Use objects, arrays, diagrams and pictorial representations, and grouping on a number line.

Arrays:

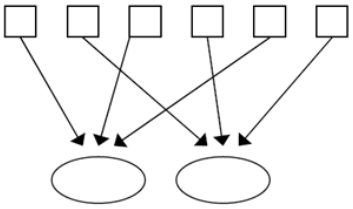


12 ÷ 3 = 4

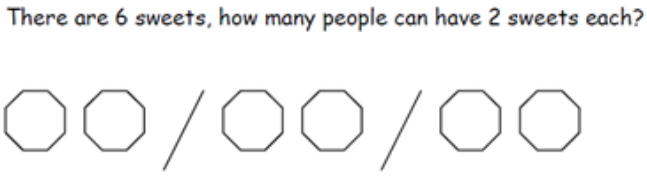
This represents **12 ÷ 3, posed as** how many groups of 3 are in 12?
Pupils should also show that the same array can represent **12 ÷ 4 = 3** if grouped horizontally.

Know and understand sharing and grouping:

6 sweets shared between 2 people, how many do they each get?



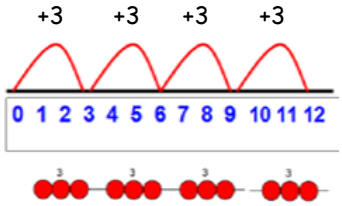
Sharing



Grouping

Children should be taught to recognise whether problems require sharing or grouping.

Grouping using a number line:



12 ÷ 3 = 4

Group from zero in equal jumps of the divisor to find out "how many groups of _ in _ ?". Pupils could and using a bead string or practical apparatus to work out problems like "A CD costs £3. How many CDs can I buy with £12?" **This is an important method to develop understanding of division as grouping.**

Pose **12 ÷ 3** as "**How many groups of 3 are in 12?**"

Key Vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, **divide, divided by, divided into, division, grouping, number line, left, left over**

Key number skills needed for division at Stage 2:

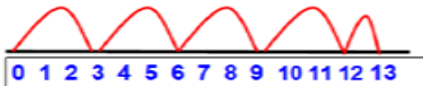
- Count in steps of 2, 3, and 5 from 0
- Recall and use multiplication and division facts for the **2, 5 and 10** multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the x, ÷ and = signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Stage 3 Divide 2-digit numbers by a single digit (where there is no remainder in the final answer)

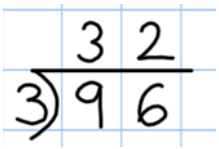
STEP 1: Children continue to work out unknown division facts by grouping on a number line from zero. They are also now taught the concept of **remainders,** as in the example. This should be introduced practically and with arrays, as well as being translated to a number line. Children should work towards calculating some basic division facts with remainders mentally for the 2s, 3s, 4s, 5s, 8s and 10s, ready for "carrying" remainders across within the short division method.

13 ÷ 3 = 4r1

+3 +3 +3 +3 r1

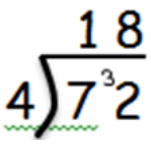


STEP 2: Once children are secure with division as grouping and demonstrate this using number lines, arrays etc., **short division** for larger 2-digit numbers should be introduced, initially with carefully selected examples requiring no calculating of remainders at all. **Remind children of correct place value, that 96 is equal to 90 and 6, but in short division, pose:**
How many 3's in 9? = 3, and record it above the **9 tens**. How many 3's in 6? = 2, and record it above the **6 units**.



Limit numbers to **NO** remainders in the answer **OR** carried (each digit must be a multiple of the divisor).

STEP 3: Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation (e.g. 96†4), and be taught to "carry" the remainder onto the next digit. **If needed, children should use the number line to work out individual division facts that occur which they are not yet able to recall mentally.**



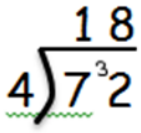
Limit numbers to **NO** remainders in the final answer, but with remainders occurring within the number being divided into

Key Vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, **inverse, short division, "carry", remainder, multiple**
Key number skills needed for division at Stage 3:

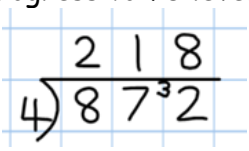
- Recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables (through doubling, connect the 2, 4 and 8s).
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- Solve problems, in contexts, and including missing number problems, involving multiplication and division.
- Pupils develop efficient mental methods, for example, using multiplication and division facts (e.g. using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (30 × 2 = 60, so 60 ÷ 3 = 20 and 20 = 60 ÷ 3).
- Pupils develop reliable written methods for division, starting with calculations of 2-digit numbers by 1-digit numbers and progressing to the formal written method of short division.

Stage 4 Divide up to 3-digit numbers by a single digit

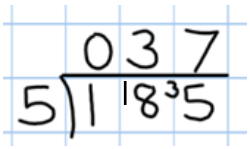
STEP 1: Pupils must be secure with the process of short division for dividing 2-digit numbers by a single digit (**those that do not result in a final remainder** —see steps in Y3), but must understand how to calculate remainders, using this to "carry" remainders within the calculation process (see example).



STEP 2: Pupils move onto dividing numbers with up to **3-digits** by a single digit, however problems and calculations provided should **not result in a final answer with remainder** at this stage. Children who exceed this expectation may progress to Y5 level.



STEP 3: When the answer for the **first column** is zero (1 ÷ 5, as in example), children could initially write a zero above to acknowledge its place, and must always "carry" the number (1) over to the next digit as a remainder.



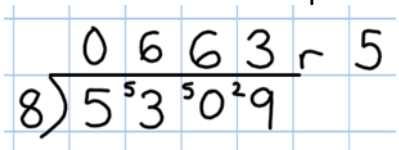
Key Vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, „carry", remainder, multiple, **divisible by, factor**

Key number skills needed for division at Stage 4:

- **Recall multiplication and division facts for all numbers up to 12 x 12.**
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number
- Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example 200 × 3 = 600 so 600 ÷ 3 = 200
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

Stage 5 Divide up to 4 digits by a single digit, including those with remainders.

Short division with remainders: Now that pupils are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where **pupils consider the meaning of the remainder and how to express it,** i.e. as a fraction, a decimal, or as a rounded number or value , depending upon the context of the problem.



The answer to 5309 ÷ 8 could be expressed as 663 and five eighths, 663 r 5, or rounded as appropriate to the problem involved.

If children are confident and accurate introduce long division for pupils who are ready to divide any number by a 2-digit number (e.g. 2678 ÷ 19). This is a Stage 6 expectation.

Key Vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, „carry", remainder, multiple, divisible by, factor, inverse, **quotient, prime number, prime factors, composite number (non-prime)**

Key number skills needed for division at Stage 5:

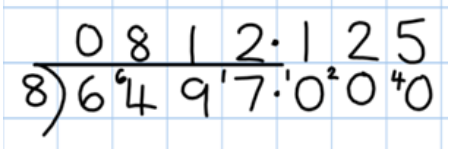
- Recall multiplication and division facts for all numbers up to 12 x 12 (as in Y4).
- Multiply and divide numbers mentally, drawing upon known facts.
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two number.
- Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Work out whether a number up to 100 is prime, and recall prime numbers to 19.
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Use multiplication and division as inverses.
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. 98 ÷ 4 = 24 r2 = 241/2 = 24.5 ≈ 25).
- Solve problems involving combinations of all four operations, including understanding of the equals sign, and including division for scaling by different fractions and problems involving simple rates.

Stage 6 Divide at least 4 digits by both single-digit and 2-digit numbers

(including decimal numbers and quantities)

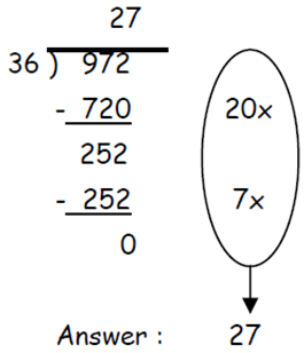
Short division with remainders: Pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.

Calculating a decimal remainder: In this example, rather than expressing the remainder as **r 1**, a decimal point is added after the units because there is still a remainder, and the one remainder is carried onto zeros after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy for the problem being solved.



Introduce long division by chunking **for dividing by 2 digits.**

Find out "How many 36s are in 972?" by subtracting "chunks" of 36, until zero is reached (or until there is a remainder).
Teach pupils to write a "**useful list**" first at the side that will help them decide what chunks to use e.g.:
1x = 36; 2x = 72; 5x = 180; 10x = 360; 100x = 3600
Introduce the method in a simple way by limiting the choice of chunks to "Can we use 10 lots? Can use 100 lots?" As children become confident with the process, encourage more efficient chunks to get to the answer more quickly (e.g. 20x, 5x), and expand on their "useful" lists.



Key Vocabulary: As previously, plus **common factor**
Key number skills needed for division at Stage 6:

- Recall and use multiplication and division facts for all numbers to 12 x 12 for more complex calculations
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long
- division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Solve problems involving all 4 operations.
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem.
- Use written division methods in cases where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specified degrees of accuracy.